Your challenge to calculate PI by hand

This is an excerpt from my other file at “engert.us/erwin/miscellaneous/William Shanks 707 digits.pdf”. Here is how you can calculate PI without a calculator or a computer. This is one of the most used formula is PI = 4 * (4 * ATN(1/5) - ATN(1/239). The function ATN(X) = X - X^3/3 + X^5/5 – X^7/7 + X^9/9 etc. can be used to calculated the ATN function. The value for X^3/3 is the same as X*X*X / 3. The process continues until the value reaches enough 0’s and no longer to affects the total sum. See the next page how it works to produce the values needed.

If you would like to see if you are able to do a small sample of what William Shanks did, but he did on a much larger scale. To do it correctly all the work must be done by one person with paper and pencil only, no calculating type device as a calculator or computer even an abacus. This would require you to know how to do long division without any aid other than your own mind plus paper and pencil. Can you do the challenge or will you just say it is too hard I am not capable to do the task, which are you. Without looking at the following page except to see if you can do the first two terms correct. The work can be done in several days if you work hard and steady and long, or use a group effort to speed up the work. There are some short cuts starting with doing the powers of 1/5, after the first term divide by 25 from sub term to sub term. In this case multiply by 4 and divide by 100 which are the same as shifting to the right by two digits. Then do the division by the term number i.e. 3, 5, 7 etc.The value of 1/5 is .2 while the next term is .2 times 4 is .8 then shift to the right by two digits which is .008, which intern has to be divided by 3 which gives you .00266666 etc.

These formulas are sometime referred to as Machin-like formula. There are many longer formulas that take more total terms to reach the same number of digits. Remember to alternate the addition and subtraction of every other term. Have fun with this problem. If you want you could use any of the following formulas listed on page 18 with further detailed information.

An early calculation not mentioned as often also had the same error as his 1873 work. The total term listing for 530 digits with 607 digits for PI is found in William Shanks 1853 calculation book “Contributions to mathematics, comprising chiefly the rectification of the circle”. If someone would like to figure out where the error is in which term has the error. The next entry I have supplied term level for PI to 720 digits. One error was found to be in the term (1/5)^497/497 he omitted a 0 in the term in the 531 digit he had 8482897 instead of 80482897, his 1853 book just missed displaying this error. This 0 would repeat at digit 741. This error only corrects an extra 38 digits, 1/497 has a repeat pattern of 210 and the (1/5)^497 fills more than the 707 digit position, the repeat starts at 708 digits. In the first series there still are more errors, and there is still an error in the second series. Due to William Shanks error all new records have to be calculated using two different methods, so his work was not in vain, even in 1947 they knew they needed to do two calculations. If you would like to see the detailed listing of each term William Shanks went through see the following web link “engert.us/erwin/miscellaneous/PI.pdf”.
Here is a term by term breakdown for the solution of computing PI to 35 digits. Note how each term is getting closer to 0. You must always calculate more digits than required to for the accuracy you desire hence 40 digits to get a 35 digit answer. The reason I picked 35 digits was that Ludolph Van Ceulen (1540 - 1610) spent most of his life working out PI to 35 decimal places. PI is sometimes known as Ludolph's Constant the value is carved on his tombstone.

\[
\begin{align*}
\frac{1}{5} &= + 0.2000000000 \\
\left(\frac{1}{5}\right)^{3/3} &= - 0.0026666666 \\
\left(\frac{1}{5}\right)^{5/5} &= + 0.0000640000 \\
\left(\frac{1}{5}\right)^{7/7} &= - 0.0000018285 \\
\left(\frac{1}{5}\right)^{9/9} &= + 0.0000000568 \\
\left(\frac{1}{5}\right)^{11/11} &= - 0.0000000018 \\
\left(\frac{1}{5}\right)^{13/13} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{15/15} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{17/17} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{19/19} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{21/21} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{23/23} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{25/25} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{27/27} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{29/29} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{31/31} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{33/33} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{35/35} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{37/37} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{39/39} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{41/41} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{43/43} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{45/45} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{47/47} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{49/49} &= + 0.0000000000 \\
\left(\frac{1}{5}\right)^{51/51} &= - 0.0000000000 \\
\left(\frac{1}{5}\right)^{53/53} &= + 0.0000000000
\end{align*}
\]

\[
\begin{align*}
\tan^{-1}\left(\frac{1}{5}\right) &= + 0.1973955598 \\
4\tan^{-1}\left(\frac{1}{5}\right) &= + 0.7895822393 \\
\frac{3}{239} &= + 0.0041840760
\end{align*}
\]

\[
\begin{align*}
4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) &= \pi/4 \\
4\left(4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)\right) &= \pi
\end{align*}
\]

The number PI correct to 40 positions after the decimal point is.

\[
\begin{align*}
\pi &= + 3.1415926535 8979323846 2643383279 5028841988
\end{align*}
\]

The total number of number of terms is 35. The error of the computed value and the correct is 17 parts in the last digit, affecting the last two digits.